الإسم = حسام الدين

اللقب = همبسي

الكراس = الإهتزازات والأمواج

الفوج = فيزياء (06)

الموسم الدراسي: 2014.2013

$$= \frac{f_0}{2an\omega}(\sin n\pi - \sin 2n\pi + \sin n\pi)$$

$$a_n = 0$$

$$b_n = \frac{1}{4a}\left[\int_0^{2a} f_0 \sin n\omega t \, dt - \int_{2a}^{4a} f_0 \sin n\omega t \, dt\right]$$

$$= \frac{-1}{4an\omega}\left[f_0 \cos n\omega t \Big|_0^{2a} - f_0 \cos n\omega t \Big|_{2a}^{4a}\right]$$

$$= \frac{-f_0}{}\left[\left(\cos 2a\, n\frac{\pi}{2a} - \cos a\right) - \left(\cos 4a\, n\frac{\pi}{2a} - \cos 2a\, n\frac{\pi}{2a}\right)\right]$$

$$= \frac{-f_0}{}\left[(\cos n\pi - 1) - (\cos 2an\pi - \cos n\pi)\right]$$

$$= \frac{f_0}{}\left[1 - \cos n\pi + \cos 2an\pi - \cos n\pi\right]$$

$$= \frac{f_0}{}\left[2 - 2\cos n\pi\right]$$

$$b_n = \frac{f_0}{n\pi}(1 - \cos n\pi)$$

$$f(t) = \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{f_0}{n\pi}(1 - \cos n\pi) \sin n\omega t$$

التمرين السابع =

$$J = J + J = \frac{1}{2} mR^2 + mR^2$$

$$= \frac{3}{2} mR^2$$

لدينا

$$J \ddot{\theta} = \sum I$$

$$\frac{3}{2} mR^2 \ddot{\theta} = -mgR \sin\theta$$

$$\frac{3}{2} mR^2 \ddot{\theta} + mgR \sin\theta = 0$$

2014.10.20

$$f(t) = a_0 + \sum_{n=1} a_n \cos \omega t + b_n \sin \omega t$$

$$a_0 = \frac{1}{T}\int_0^T f(t)$$

$$a_n = \frac{1}{T}\int_0^T f(t) \cos n\omega t$$

$$b_n = \frac{1}{T}\int_0^T f(t) \sin n\omega t \, dt$$

السلسلة (01) =

التمرين 08 =

$$a_0 = \frac{1}{4a}\left[\int_0^{2a} f_0 \, dt + \int_0^{4a} -f_0 \, dt\right]$$

$$= \frac{1}{4a}\left[f_0 2a - f_0 2a\right] = 0$$

$$a_n = \frac{1}{4a}\left[\int_0^{2a} f_0 \cos n\omega t \, dt + \int_{2a}^{4a} -f_0 \cos n\omega t \, dt\right]$$

$$= \frac{1}{4a}\left[\left[f_0 \frac{1}{n\omega} \sin n\omega t\right]_0^{2a} + \left[-f_0 \frac{1}{n\omega} \sin n\omega t\right]_{2a}^{4a}\right]$$

$$= \frac{1}{4a}\left[\left[\frac{1}{n\omega} f_0 (\sin 2an\omega)\right] - \left[\frac{1}{n\omega} f_0 (\sin 4an\omega)\right] \sin 2an\omega\right]$$

$$= \frac{1}{4an\omega} \cdot f_0 \left(\sin 2an\frac{\pi}{2a} - \sin 4an\frac{\pi}{2a} + \sin 2an\frac{\pi}{2a}\right)$$

2014.10.26

السلسلة (2)

التمرين الأول:

الشكل (1):

1- إيجاد الطاقة الحركية:

$$T = T_m + T_\mu + T_m$$

$$T_m = \frac{1}{2} J_m \dot{\theta}^2 \quad , \quad J_m = \frac{1}{2} m R^2$$

$$= \frac{1}{4} m R^2 \dot{\theta}^2$$

$$T_\mu = \frac{1}{2} J_\mu \dot{\theta}^2 \quad , \quad J_\mu = \mu d^2$$

$$= \frac{1}{2} \mu d^2 \dot{\theta}^2$$

$$T_m = \frac{1}{2} m \dot{x}_m^2 = \frac{1}{2} m R^2 \dot{\theta}^2$$

$$T = \frac{1}{4} m R^2 \dot{\theta}^2 + \frac{1}{2} \mu d^2 \dot{\theta}^2 + \frac{1}{2} m R^2 \dot{\theta}^2$$

- إيجاد الطاقة الكامنة:

$$U = U_m + U_\mu$$

$$U_m = -\int_0^{R\theta} m g \, dy = -m g R \theta$$

$$U_\mu = -\int_{d\cos\theta_0}^{d\cos(\theta_0+\theta)} \mu g \, dy = -\mu g \left[d\cos(\theta_0+\theta) - d\cos\theta_0 \right]$$

$$= \mu g d \left[\cos\theta_0 - \cos\theta \right]$$

$$= \mu g d \left(\frac{\theta^2}{2} \cos\theta_0 + \theta \sin\theta_0 \right)$$

$$U = -m g R \theta + \mu g d \left(\frac{\theta^2}{2} \cos\theta_0 + \theta \sin\theta_0 \right)$$

2- شرط التوازن:

$$\frac{\partial U}{\partial \theta} = 0 \qquad \theta = 0$$

$$\frac{\partial U}{\partial \theta} = -m g R + \mu g d \sin\theta_0 = 0$$

بما أن الاهتزازات صغيرة يكون:

$$\sin\theta \simeq \theta \quad , \quad \cos\theta = 1 - \frac{\theta^2}{2}$$

$$\frac{3}{2} m R^2 \ddot{\theta} + m g R \theta = 0$$

وهذه هي المعادلة التفاضلية للحركة.

ملاحظة:

$$\ddot{\theta} + a\theta = 0$$

لها حل من الشكل

$$\theta(t) = A \sin(\sqrt{a}\, t + \varphi)$$

$$= A \sin\sqrt{a}\, t + B \cos\sqrt{a}\, t$$

3- لدينا:

$$\omega = \sqrt{\frac{2g}{3R}} \Rightarrow$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{3R}}} = \sqrt{\frac{4\pi^2 \, 3R}{2g}}$$

$$= \sqrt{\frac{6\pi^2 R}{g}}$$

$$T = \sqrt{\frac{6 \cdot 0,2 \cdot \pi^2}{10}} = \frac{2\pi}{10}\sqrt{3} = \frac{\pi}{5}\sqrt{3}\ s$$

$$\theta(0) = A \sin(\varphi) = 9$$

$$\dot{\theta}(0) = A \sqrt{\frac{2g}{3R}} \cos(\varphi) = 0$$

$$\Rightarrow \cos(\varphi) = 0 \quad , \quad \varphi = \frac{\pi}{2}$$

$$A \sin\frac{\pi}{2} = 9^\circ \Rightarrow A = 9^\circ$$

$$\theta(t) = 9 \sin\left(\sqrt{\frac{2g}{3R}}\, t + \frac{\pi}{2} \right)$$

$$= 9 \left(\sin\omega t \cdot \cos\frac{\pi}{2} + \cos\omega t \sin\frac{\pi}{2} \right)$$

$$= 9 \cos\omega t$$

$$= 9 \cos\sqrt{\frac{2g}{3R}}\, t \quad , \quad \dot{\theta} = 9\sqrt{\frac{2g}{3R}}$$

$$\sin\sqrt{\frac{2g}{3R}}\, t = -\sqrt{\frac{14\,g}{R}} \ldots \omega t$$

تابع التمرين 7 =

$$\dot\theta\left(\frac{T}{8}\right) = -\sqrt{\frac{54g}{R}}\sin\left(\frac{2\pi}{T}\cdot\frac{T}{8}\right)$$

$$\dot\theta\left(\frac{T}{8}\right) = \sqrt{54\frac{g}{R}}\cdot\sin\frac{\pi}{4}$$

$$= -\sqrt{54\frac{g}{R}}\cdot\frac{\sqrt{2}}{2} = -\sqrt{27\frac{g}{R}}\ Rad/s$$

$$V_{max} = R/2\ \dot\theta_{max}\quad \frac{R}{2}\cdot 9\sqrt{\frac{2g}{3R}}$$

$$= \sqrt{\frac{27}{2}Rg}\quad m/s$$

التمرين السادس =

$$\langle x\rangle = \frac{1}{T_0}\int_0^T x\cdot dt$$

$$= \frac{1}{T}\int_0^T A\sin(\omega t+\varphi)\cdot dt$$

$$= -\frac{1}{T_0}\frac{A}{\omega}\cos(\omega t+\varphi)\Big|_0^T$$

$$= -\frac{A}{2\pi}\left[\cos\left(\frac{2\pi}{T}T+\varphi\right)-\cos\varphi\right]$$

$$= -\frac{A}{2\pi}\left[\cos(2\pi+\varphi)-\cos\varphi\right]$$

($\cos\varphi$, 0)

$$\langle x\rangle = 0$$

$$\langle x^2\rangle = 1/T_0\int_0^T x^2\,dt$$

$$= 1/T\int_0^T A^2\sin^2\alpha\cdot dt$$

$$= 1/T\,A^2\sin^2(\omega t+\varphi)\cdot dt$$

$$= \frac{A^2}{2T}\int_0^T\left(\frac{1-\cos(2\omega t+2\varphi)}{2}\right)dt$$

$$= \frac{A^2}{2T}\left(\int_0^T dt - \int_0^T\cos(2\omega t+2\varphi)dt\right)$$

-4

شرط الاهتزاز:

$$\frac{\partial^2 U}{\partial\theta^2} > 0$$

$$\frac{\partial^2 U}{\partial\theta^2} = Mgd\cos\theta_0 > 0$$

3- المعادلة التفاضلية للحركة:

$$L = T - U$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \frac{\partial L}{\partial\theta} = 0 \quad (1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) = 1/2\,(mR^2+Md^2+mR^2)\ddot\theta$$

$$\frac{\partial L}{\partial\theta} = -(-mgR+Mgd(\theta\cos\theta_0+\sin\theta_0))$$

$$(1)\Leftrightarrow\left(\frac{m}{2}R^2+Md^2+mR^2\right)\ddot\theta + (-mgR+Mgd(\theta\cos\theta_0+\sin\theta_0)) = 0$$

وبالتالي:

$$\left(\frac{m}{2}R^2+Md^2+mR^2\right)\ddot\theta + Mgd\cos\theta_0\,\theta = 0$$

$$\ddot\theta + \frac{Mgd\cos\theta_0}{\frac{m}{2}R^2+Md^2+mR^2}\theta = 0$$

وهي من الشكل:

$$\ddot\theta + a\theta = 0$$

معادلة الحركة:

$$\theta(t) = A\sin(\omega t+\varphi)$$

$$\omega = \sqrt{\frac{Mgd\cos\theta_0}{\frac{m}{2}R^2+Md^2+mR^2}}$$

$\varphi = 41,81$; $\varphi' = 138,19$

$\dot{x}(0) = 0,03 \cdot 20\pi \cos(41,81)$

$x(t) = 0,03 \cdot \sin(20\pi + 0,96\pi)$

2014.11.09

تابع ت 1 :

الشكل (2) :

1- ايجاد الطاقة الحركية :

$T = T_{3m} + T_m$

$T_{3m} = \frac{1}{2} J \dot{\theta}^2$

$J = \frac{1}{12} 3mL^2 + 3m(\frac{L}{2})^2$

$J = \frac{1}{4} mL^2 + 3m \cdot \frac{L^2}{4} = mL^2$

$T_{3m} = \frac{1}{2} mL^2 \dot{\theta}^2$

$T_m = \frac{1}{2} J \dot{\theta}^2$

$J = m(\sqrt{2}L)^2 = 2mL^2$

$T_m = mL^2 \dot{\theta}^2$

$T = \frac{1}{2} mL^2 \dot{\theta}^2 + mL^2 \dot{\theta}^2$

$= \frac{3}{2} mL^2 \dot{\theta}^2$

- ايجاد الطاقة الكامنة :

$U = U_{3m} + U_m + U_k + U_{k'}$

$U_{3m} = -\int_0^{\sin\theta} 3mg\, dy$

$= -3mg(-\frac{L}{2}\sin\theta - 0)$

$= \frac{3}{2} mgL$

$U_m = -\int_{-L}^{-\sqrt{2}L\sin(\frac{\pi}{4}+\theta)} mg\, dy$

$= -mg[-\sqrt{2}L\sin(\frac{\pi}{4}+\theta) + L]$

$= -mgL[-\sqrt{2}\sin(\frac{\pi}{4}+\theta) + \cos\frac{\pi}{4} \cdot \sin\theta + 1$

$= -mgL[-\cos\theta - \sin\theta + 1]$

$= \frac{A^2}{2T}(T - 0) = \frac{A^2}{2}$

التمرين الخامس :

- التفرع جمع مباشر

- التسلسل جمع مقلوب .

التمرين الرابع :

1- التواتر الأعظمي يوافق القوة الأعظمية :

$F_{max} = m\ddot{x}_{max}$

$\ddot{x}_{max} = A\omega^2$

$3cm = 0,03\, m$, $f = 1\, Hz$

$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi\, Rad/s$

$\ddot{x} = 0,03 \cdot 4\pi^2 = 0,12\pi^2\, m/s^2$

$F_{max} = m\ddot{x} = 10 \cdot 0,12 = 1,2\pi^2\, N$

2- $\dot{x} = A\omega = 0,03 \cdot 2\pi$

$= 0,06\pi\, m/s$

التمرين الثالث :

$x(t) = A\sin(\omega t + \varphi)$

$A = 0,03\, m$

$f = 10\, Hz$ $\quad \omega = 2\pi f = 20\pi\, Rad/s$

$x(t) = 0,03 \sin(20\pi t + \varphi)$

$\dot{x} = 0,03 \cdot 20\pi \cos(20\pi t + \varphi)$

$$\begin{cases} x(0) = 0,03 \sin\varphi = 0,02 \\ \dot{x} < 0 \end{cases}$$

$\sin\varphi = \frac{0,02}{0,03} =$

فمعادلة تفاضلية حلها جيبي من الشكل

$\theta(t) = A\sin(\omega t + \varphi)$

$\omega = \sqrt{\dfrac{-mg + KL + 2K'L}{3mL}}$

التمرين الثالث:

1- إيجاد الطاقة الحركية:

$T_M = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2$

$= \frac{1}{2} MR^2 \dot{\theta}^2 + \frac{1}{4} MR^2 \dot{\theta}^2$

$= \frac{3}{4} MR^2 \dot{\theta}^2$

$T_{m_1} = \frac{1}{2} m_1 V_{m_1}^2 = \frac{1}{2} m_1 (R\dot{\theta} - L_1 \dot{\theta})^2$

$T_{m_2} = \frac{1}{2} m_2 (R\dot{\theta} - L_2 \dot{\theta})^2$

$T = \frac{3}{4} MR^2 \dot{\theta}^2 + \frac{1}{2} m_1 (R\dot{\theta} - L_1\dot{\theta})^2$
$+ \frac{1}{2} m_2 (R\dot{\theta} - L_2\dot{\theta})^2$

$\vec{Om_1} = \vec{OO'} + \vec{O'm_1} = -R\theta \vec{i}$
$+ L_1 \sin(\theta_0 + \theta)\vec{i'} - L_1\cos(\theta_0+\theta)\vec{j'}$

$= ~~(\ldots + L_1\sin(\theta_0+\theta)\vec{i'} - L_1\cos(\theta_0+\theta)\vec{j})~~$

$\vec{V}_{m_1} = ?$

$\vec{V}^2_{m_1}$

- حساب الطاقة الكامنة:

$U_{m_1} = \int_{L_1\cos\theta_0}^{L_1\cos(\theta_0+\theta)} m_1 g\, dy = -m_1 g L_1 [\cos(\theta_0+\theta)$
$- \cos\theta_0]$

$(1 - \frac{\theta^2}{2})$ θ

$= -m_1 g L_1 [\cos\theta_0 . \cos\theta - \sin\theta_0 . \sin\theta$
$- \cos\theta_0]$

$= -mgL_1 [-\frac{\theta^2}{2}\cos\theta_0 - \theta\sin\theta_0]$

$= mgL_1 [\frac{\theta^2}{2}\cos\theta_0 + \theta\sin\theta_0]$

$= -mgL[-(1 - \frac{\theta^2}{2}) - \theta + 1]$

$= -mgL[\frac{\theta^2}{2} - \theta]$

$= mgL[\theta - \frac{\theta^2}{2}]$

$U_K = \frac{1}{2} K(\Delta x + L\theta)^2$

$U_{K'} = \frac{1}{2} K'(\Delta x + \sqrt{2}L\theta)^2$

$U = \frac{3}{2} mgL\theta + mgL(\theta - \frac{\theta^2}{2})$
$+ \frac{1}{2}K(\Delta x + L\theta)^2 + \frac{1}{2}K'(\Delta x + \sqrt{2}L\theta)^2$

2- شرط التوازن:

$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0$

$\frac{\partial U}{\partial \theta} = \frac{3}{2} mgL + mgL[1 - \theta] + K(\Delta x + L\theta)L + K'(\Delta x + \sqrt{2}L\theta)\sqrt{2}L$

$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0 \Leftrightarrow \frac{3}{2} mgL + mgL + KL\Delta x + \sqrt{2}LK'\Delta x = 0$

- شرط الاهتزاز:

$\frac{\partial^2 U}{\partial \theta^2} > 0 \Rightarrow -mgL + KL^2 + 2K'L^2 > 0$

3- إيجاد معادلة الحركة:

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) + \frac{\partial U}{\partial \theta} = 0$

$= 3mL^2\ddot{\theta} + \frac{3}{2} mgL + mgL - mgL$
$+ K\Delta x L + KL^2\theta + K'\Delta x\sqrt{2}L$
$+ 2K'L^2\theta = 0$

$3mL^2\ddot{\theta} + (-mgL + KL^2 + 2K'L^2)\theta = 0$

$\ddot{\theta} + \frac{-mgL + KL^2 + 2K'L^2}{3mL^2}\theta = 0$

$m_1 g L_1 \theta \sin\theta_0 - m_2 g L_2 \cos\theta_0 = 0$

$+ m_2 g L_2 \theta \sin\theta_0 - m_2 g L_2 \cos\theta_0 = 0$

$[\frac{3}{2} m R^2 + m_1 (R - L_1)^2 + m_2 (R - L_2)^2] \ddot{\theta}$

$+ [m_1 g L_1 \cos\theta_0 + m_2 g L_2 \sin\theta_0] \theta = 0$

$[3m^2 + 2mR^2 + 4mR^2] \ddot{\theta} + [4mgR \cos\theta_0 + 3mgR \sin\theta_0] \theta = 0$

$9mR^2 \ddot{\theta} + (4\cos\theta_0 + 3\sin\theta_0) mgR\theta = 0$

$$\ddot{\theta} + \frac{4\cos\theta_0 + 3\sin\theta_0}{9} \frac{g}{R} \theta = 0$$

$\theta(t) = A \sin(\omega t + \varphi)$

$$\omega = \sqrt{\frac{4\cos\theta_0 + 3\sin\theta_0}{9} \frac{g}{R}}$$

$\theta(0) = \pi/6 \quad \begin{cases} A \sin\varphi = \pi/6 \\ A\omega \cos\varphi = 0 \end{cases}$

$\dot{\theta}(0) = 0$

$\cos\varphi = 0$

$\varphi = (2n + \pi/2)$

$n = 0$

$\varphi = \pi/2 \qquad A = \pi/6$

2014.11.23

التمرين الخامس:

1- ايجاد المعادلة التفاضلية لحركة الشحنة

$V_L + V_C = 0$

$V_L = L \, di/dt$

$V_L = L \frac{d^2 q}{dt}$ ولدينا: $i = dq/dt$

$V_C = 1/C \, q$

$L\ddot{q} + q/C = 0$

$$\ddot{q} + \frac{1}{LC} q = 0$$

$$U_{m_2} = \int_{L_2 \sin\theta}^{L_2 \sin(\theta_0+\theta)} m_2 g \, dy = -m g L_2 (\sin(\theta_0 + \theta) - \sin\theta_0)$$

$1 - \frac{\theta^2}{2}$

$= -m_2 g L_2 [\sin\theta_0 \cos\theta + \cos\theta_0 \sin\theta - \sin\theta_0]$

$= -m_2 g L_2 [-\frac{\theta^2}{2} \sin\theta_0 + \theta \cos\theta_0]$

$= m_2 g L_2 [\frac{\theta^2}{2} \sin\theta_0 - \theta \cos\theta_0]$

$U = m_1 g L_1 [\frac{\theta^2}{2} \cos\theta_0 + \theta \sin\theta_0] + m_2 g L_2 [\frac{\theta^2}{2} \sin\theta_0 - \theta \sin\theta_0]$

2- ايجاد شرط التوازن:

$$\frac{\partial U}{\partial \theta} = m_1 g L_1 [\theta \cos\theta_0 + \sin\theta_0] + m_2 g L_2 [\theta \sin\theta_0 - \cos\theta_0]$$

$$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0 \Leftrightarrow m_1 g L_1 \sin\theta_0 - m_2 g L_2 \cos\theta_0 = 0$$

- ايجاد مقدار الزاوية θ_0 عند التوازن:

$m_1 g L_1 \sin\theta_0 = m_2 g L_2 \cos\theta_0$

$4 m R \sin\theta_0 = 3 m R \cos\theta_0$

$$\tan\theta_0 = \frac{\sin\theta_0}{\cos\theta_0} = 3/4 \Rightarrow \theta_0 = \arctan 3/4$$

$= 36{,}86°$

3- شرط الاهتزاز:

$$\frac{\partial^2 U}{\partial \theta^2} > 0 \Rightarrow m_1 g L_1 \cos\theta_0 + m_2 g L_2 \sin\theta_0$$

النظام دوما يهتز

4- ايجاد معادلة الحركة:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) + \left(\frac{\partial U}{\partial \theta}\right) = 0 \Longleftrightarrow \frac{3}{2} m R^2 \ddot{\theta}$$

$+ m_1 (R - L_1)^2 \ddot{\theta} + m_2 (R - L_2)^2 \ddot{\theta}$

$+ m_1 g L_1 \theta \cos\theta_0 + m_1 g L_1 \sin\theta_0$

$= -mg\frac{L}{3}(1-\cos\theta)$

$= -mg\frac{L}{3}(1-(1-\frac{\theta^2}{2}))$

$= -mg\frac{L}{3}(+\frac{\theta^2}{2}) = mg\frac{L}{6}\theta^2$

$U_{2m} = -\int_{2L/3}^{2L/3\cos\theta} 2mg\, dy$

$= -2mg(\frac{2}{3}L\cos\theta - \frac{2}{3}L)$

$= -2mg\frac{2}{3}L(\cos\theta - 1)$

$= -\frac{4}{3}mgL(1-\frac{\theta^2}{2}-1) = \frac{2}{3}mgL\theta^2$

$U_{K_1} = \frac{1}{2}K_1(\Delta x - \frac{L}{6}\theta)^2$

$U_{K_2} = \frac{1}{2}K_2(\Delta x - \frac{L}{3}\theta)^2$

$U = mg\frac{L}{6}\theta^2 + \frac{2}{3}mgL\theta^2$

$+\frac{1}{2}K_1(\Delta x - \frac{L}{3}\theta)^2 + \frac{1}{2}K_2(\Delta x - \frac{L}{3}\theta)^2$

$L = T - U$

$= \frac{1}{2}mL^2\dot\theta^2 - (\frac{1}{2}mL\theta^2 + \frac{1}{2}K\frac{L^2}{36}\theta^2$

$+\frac{1}{2}K_2\frac{L^2}{9}\theta^2)$

$\frac{d}{dt}(\frac{\partial L}{\partial\dot\theta}) - \frac{\partial L}{\partial\theta} = \frac{-\partial D}{\partial\dot\theta}$ / $D = \frac{1}{2}\alpha\dot x_c^2$

$x_c = \frac{L}{2}\theta \Rightarrow \dot x_c = \frac{L}{2}\dot\theta$

$D = \frac{1}{2}\alpha\frac{L^2}{2}\dot\theta^2$

$mL^2\ddot\theta + mL\theta + K\frac{L^2}{36}\theta + K_2\frac{L^2}{9}\theta$

$= \alpha\frac{L^2}{4}\dot\theta$

$mL^2\ddot\theta + \alpha\frac{L^2}{4}\dot\theta + (mgL + \frac{1}{36}K_1L^2 + \frac{1}{9}$

$K_2L^2)\theta = 0$

$K_2 = \frac{mg}{L}$ $K_1 = \frac{32mg}{L}$

$mL^2\ddot\theta + \alpha\frac{L^2}{4}\dot\theta + (mgL + \frac{1}{36}\frac{32mg}{L}$

$L^2 + \frac{1}{9}\frac{mg}{L})\theta = 0$

$\omega = \sqrt{\frac{1}{LC}}$

$\omega = \frac{1}{\sqrt{40.10^{-3}.10^{-6}}} = 5.10^3 \frac{Rad}{s}$

$f = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi} = \frac{5.10^3}{2\pi}$

$= 2,5\pi^{-1}.10^3 Hz$

- ايجاد حل المعادلة التفاضلية:

$q(t) = A\sin(\omega t + \varphi)$

$= A\sin(5.10^3 t + \varphi)$

$i(t) = 5.10^3 A\cos(5.10^3 t + \varphi)$

عند $t = 0$

$q(0) = q_0$

$i(0) = 0$

السلسلة (03):

التمرين الأول:

$T = T_m + T_{2m}$

$T_m = \frac{1}{2}J\dot\theta^2$ / $J = m(\frac{L}{3})^2$

$= \frac{1}{18}mL^2\dot\theta^2$

$T_{2m} = \frac{1}{2}J\dot\theta^2$ / $J = 2m\frac{4L^2}{9}$

$= \frac{4}{9}mL^2\dot\theta^2$

$T = \frac{1}{18}mL^2\dot\theta^2 + \frac{4}{9}mL^2\dot\theta^2 =$

$\frac{9}{18}mL^2\dot\theta^2 = \frac{1}{2}mL^2\dot\theta^2$

$U = U_{K_1} + U_{K_2} + U_m + U_{2m}$

$U_m = -\int_{-L/3}^{-L/3\cos\theta} mg\, dy = -mg(-\frac{L}{3}\cos\theta + \frac{L}{3}$

$\delta^2 . 4\pi^2 = 10^{-2}(\omega^0 - \delta^2)$

$\delta^2 . 4\pi^2 = 10^{-2}\omega_0 - 10^{-2}\delta^2$

$\delta^2(4\pi^2 + 10^{-2}) = 10^{-2}\omega_0$

$\frac{\alpha^2}{64m^2}(4\pi^2 + 10^{-2}) = 10^{-2} . \frac{2g}{L}$

$\frac{\alpha^2}{64m^2} 2\pi^2 = 10^{-2} \, g/L$

$\alpha^2 = 10^{-2} \, g/L . \frac{64m^2}{2\pi^2}$

$\alpha = \frac{10^{-1} . 8 . m}{\pi}\sqrt{\frac{g}{2L}}$ $\frac{kg}{\ }$

$\alpha = 11.30$

$\theta(t) = C . e^{-\delta t}\sin(\omega_a t + \varphi)$

$\dot{\theta}(t) = -\delta C . e^{-\delta t}\sin(\omega_a t + \varphi)$

$+ C\omega_a e^{-\delta t}\cos(\omega_a t + \varphi)$

$\theta(0) = 0$

$\dot{\theta}(0) = -4$

$\theta(0) = C . \sin\varphi = 0$

$\dot{\theta}(0)\ -\delta . C\sin\varphi + C\omega_a\cos\varphi = -4$

$\sin\varphi = 0 \Rightarrow \varphi = n\pi$

$\varphi = \pi$ نأخذ $n = 1$

$-C\omega_a = -4$

$C = 4/\omega_a$

$\theta(t) = 4/\omega_a \, e^{-\delta t}\sin(\omega_a t + \pi)$

$\theta(t) = \frac{-4}{\omega_a} e^{-\delta t}\sin\omega_a t$

$mL^2\ddot{\theta} + \alpha\frac{L^2}{4}\dot{\theta} + mgL(1 + \frac{8}{9} + \frac{1}{9})\theta = 0$

$mL^2\ddot{\theta} + \alpha L^2/4\,\dot{\theta} + 2mgL\theta = 0$

$\ddot{\theta} + \frac{\alpha}{4m}\dot{\theta} + \frac{2g}{L}\theta = 0$

$\ddot{\theta} + 2\delta\dot{\theta} + \omega_0^2\theta = 0$

$\Delta' < 0 \Rightarrow \delta^2 - \omega_0^2 < 0$

$\Rightarrow \delta^2 < \omega_0^2 \Rightarrow \frac{\alpha^2}{64m^2} < \frac{2g}{L}$

$\alpha^2 < \frac{64m^2 . 2g}{L}$

$\Rightarrow \alpha < 8m\sqrt{2g/L}$

$\theta(t) = Ce^{-\delta(t)}\sin(\sqrt{\omega_0^2 - \delta^2}\,t + \varphi)$

$= Ce^{-\delta t}\sin(\omega_a t + \varphi)$

حيث : $\omega_a = \sqrt{\omega_0^2 - \delta^2}$

- إيجاد ثابت التخامد α :

$\frac{\theta_4}{\theta_7} = e^{0,3}$

$\theta_4(t_4) = Ce^{-\delta t_4}\sin(\omega_a t_4 + \varphi)$

$\theta_7(t_7) = Ce^{-\delta t_7}\sin(\omega_a t_7 + \varphi)$

$= Ce^{-\delta(t_4 + 3T_a)}\sin(\omega_a(t_4 + 3t_a) + \varphi)$

$= Ce^{-\delta t_4} . e^{-\delta 3t_4} \sin(\omega_a t_4 + 6\pi + \varphi)$

$= Ce^{-\delta t_4} . e^{-3\delta t_a}\sin(\omega_a t_4 + \varphi)$

$\frac{\theta_4}{\theta_7} = \frac{1}{e^{-3\delta t_a}} = e^{3\delta t_a} = e^{0,3}$

$3\delta t_a = 0,3 \Rightarrow \delta t_a = 0,1$

$\delta\frac{2\pi}{\sqrt{\omega_0^2 - \delta^2}} = 0,1$

$\delta 2\pi = 0,1\sqrt{\omega^0 - \delta^2}$

$i(0) = \omega_0 A \cos \varphi = 0$ — (**)

(**) $\Rightarrow \cos\varphi = 0 \Rightarrow \varphi = (2n+1)\pi/2$

$\varphi = \pi/2$ بأخذ $n=0$

$A = q_0/1 \qquad A = q_0$

$q(t) = q_0 e^{-\delta t} \sin(\omega_0 t + \pi/2)$

$i(t) = \omega_0 q_0 e^{-\delta t} \cos(\omega_0 t + \pi/2)$

$q(t) = q_0 e^{-\delta t} \cos \omega_0 t$

$i(t) = \omega_0 q_0 e^{-\delta t} \sin \omega_0 t$

ب) $E(t) = \frac{1}{2} L \dot{q}^2 + \frac{1}{2}\frac{1}{C} q^2$

$= \frac{1}{2} L \omega_0^2 q_0^2 e^{-2\delta t} \sin^2 \omega_0 t$

$+ \frac{1}{2C} q_0^2 e^{-2\delta t} \cos^2 \omega_0 t \qquad \omega_0 = \frac{1}{LC}$

$E(t) = \frac{1}{2} q_0^2 e^{-2\delta t} (L \omega_0 \sin^2 \omega_0 t$

$+ \frac{1}{C} \cos^2 \omega_0 t)$

$E(t) = \frac{1}{2C} q_0^2 e^{-2\delta t}$

جـ) $W(t) = E(0) - E(t)$

$= \frac{1}{2C} q_0^2 - \frac{1}{2C} q_0^2 e^{-2\delta t}$

الاستنتاج:

$\delta W(T) = E(t) - E(t+T)$

$= \frac{1}{2C} q_0^2 e^{-2\delta t} - \frac{1}{2C} q_0^2 e^{-2\delta(t+T)}$

$W(T) = \frac{1}{2C} q_0^2 e^{-2\delta t} (1 - e^{-2\delta T})$

$= E(t)(1 - e^{-2\delta \frac{2\pi}{\omega_0}})$

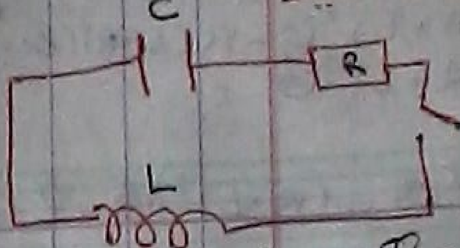
$= E(t)(1 - e^{-\frac{2\pi}{Q}})$

$Q \gg 1$ ولدينا: $x \ll 1$

$e^{x} = 1 + x$ ، تايلور

$W(T) = E(t)(1 - (1 - \frac{2\pi}{Q}))$

التمرين الثاني:



أ- Q: معامل الجودة: $Q = \frac{\omega_0}{2\delta}$

$L\ddot{q} + R\dot{q} + \frac{1}{C} q = 0$

$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{CL} q = 0$

$\ddot{q} + 2\delta \dot{q} + \omega_0^2 q = 0$ — (1)

$2\delta = R/L \quad / \quad \omega_0^2 = \frac{1}{C.L}$

$\Delta = \delta^2 - \omega_0^2$

$Q \gg 1 \Rightarrow \omega_0 \gg \delta$

$\Delta' < 0$

حل المعادلة التفاضلية (1) هو:

ب- $q(t) = A e^{-\delta t} \sin(\omega_a t + \varphi)$

حيث $\omega_a = \sqrt{\omega_0^2 - \delta^2}$

أي $\omega_a = \omega_0$

$q(t) = A e^{-\delta t} \sin(\omega_0 t + \varphi)$

لدينا: $i(t) = \frac{dq}{dt}$

$i(t) = -\delta A e^{-\delta t} \sin(\omega_0 t + \varphi)$

$+ \omega_0 A e^{-\delta t} \cos(\omega_0 t + \varphi)$

$= \omega_0 A e^{-\delta t} \cos(\omega_0 t + \varphi)$

$q(0) = q_0$

$i(0) = 0$

$q(0) = A \sin\varphi = q_0$ — (*)

$$W(T) = E(t) \cdot \frac{2\pi}{\varphi}$$

السلسلة رقم (04):

التمرين الأول: (1)

$$T_m = \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2} m V_m^2$$
$$= \frac{1}{4} M R^2 \dot{\theta}^2 + \frac{1}{2} M R^2 \dot{\theta}^2$$
$$= \frac{3}{4} M R^2 \dot{\theta}^2$$

$$T_m = \frac{1}{2} m (R\dot{\theta} + L\dot{\theta})^2$$

$$T = \frac{3}{4} M R^2 \dot{\theta}^2 + \frac{1}{2} m (L\dot{\theta} + R\dot{\theta})^2$$

$$U_m = -\int_{-L}^{-L\cos\theta} mg\, dy$$
$$= -mg(-L\cos\theta + L)$$
$$= mg\left(\frac{\theta^2}{2}\right)$$

$$U_{k'} = \frac{1}{2} k' (R\theta + L\theta)^2$$
$$= \frac{1}{2} k' (R+L)^2 \theta^2$$

$$U_k = \frac{1}{2} k (x - (R+L)\theta)^2$$

$$U = -mg\frac{\theta^2}{2} + \frac{1}{2} k'(R+L)^2 \theta^2 + \frac{1}{2} k (x - (R+L)\theta)^2$$

$$D = \frac{1}{2} f \dot{x}_m^2$$
$$= \frac{1}{2} f (R\dot{\theta} + L\dot{\theta})^2$$
$$= \frac{1}{2} f (R+L)^2 \dot{\theta}^2$$

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\theta}}\right] - \frac{\partial U}{\partial \theta} = -\frac{\partial D}{\partial \dot{\theta}}$$

لأن النظام يسير في اتجاه واحد

$$= \frac{3}{2} M R^2 \ddot{\theta} + m(R+L)^2 \ddot{\theta} - mgL\theta + k'(R+L)^2\theta - R(L+R)(x - (L+R)\theta)$$
$$= -f(R+L)^2\dot{\theta}$$

$$\left(\frac{3}{2} M R^2 + m(R+L)^2\right)\ddot{\theta} + f(R+L)^2\dot{\theta} + (-mgL + k'(R+L)^2 + k(L+R)^2)\theta$$
$$= k(R+L)x$$

$$32 m R^2 \ddot{\theta} + 16 f R^2 \dot{\theta} + 3 mg R\theta$$
$$= 4 k R B \sin \Omega t$$

$$\ddot{\theta} + \frac{1}{2}\frac{f}{m}\dot{\theta} + \frac{3}{32}\frac{g}{R}\theta = \frac{k}{mR} B \sin \Omega t$$

$$\ddot{\theta} + 2\delta\dot{\theta} + \omega_0^2\theta = B' \sin \Omega t \quad (1)$$

$$2\delta = \frac{1}{2}\frac{f}{m}, \quad \omega_0^2 = \frac{3}{32}\frac{g}{R}, \quad B' = \frac{L}{8}\frac{k}{mR} B$$

$$\theta(t) = \theta_c(t) + \theta_p(t)$$

نهتم بإيجاد الحل الخاص باعتباره يحوي النبض Ω في النمط الدائم

$$\theta_p(t) = A \sin(\Omega t + \varphi)$$

لتسهيل الحساب نستبدل في المعادلة (1) $\sin \Omega t$ بـ $e^{j(\Omega t)}$

$$(1) \Rightarrow \ddot{\theta} + 2\delta\dot{\theta} + \omega_0\theta = B' e^{j\Omega t}$$

$$\Rightarrow \theta_p(t) = A e^{j(\Omega t + \varphi)}$$
$$\theta_p'(t) = j\Omega A e^{j(\Omega t + \varphi)}$$
$$\theta_p''(t) = -\Omega^2 A e^{j(\Omega t + \varphi)}$$

بالتعويض نجد

$$(-\Omega^2 + 2\delta j\Omega + \omega_0^2) A e^{j(\Omega t + \varphi)} = B' e^{j\Omega t}$$

$$[(\omega_0^2 - \Omega^2)A + j2\delta\Omega A] e^{j\varphi} = B'$$

$$[(\omega_0^2 - \Omega^2)A + j2\delta\Omega A] = B' e^{-j\varphi}$$

$\Leftrightarrow \ddot\theta + \frac{f}{m}\dot\theta + \frac{1}{mL^2}(mgL + \frac{1}{4}KL^2)\theta$
$= \frac{L}{mL^2}F_0 \sin \Omega t$

$\Leftrightarrow \ddot\theta + 2\delta\dot\theta + \omega_0^2\theta = \frac{m}{L}F_0 \sin \Omega t$

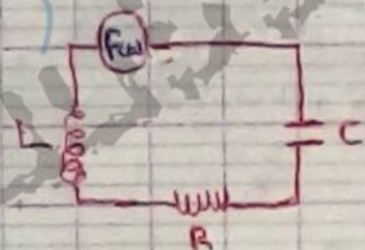
$$\ddot\theta + \frac{f}{m}\dot\theta + \frac{gm + \frac{K}{4}L}{mL}\theta = \frac{F_0}{mL}\sin \Omega t$$

$$\ddot\theta + 2\delta\dot\theta + \omega_0^2\theta = F_0' \sin \Omega t$$

$2\delta = \frac{f}{m}$, $\omega_0^2 = \frac{gm + \frac{K}{4}L}{mL}$, $F_0' = \frac{F_0}{mL}$

[illegible]

التمرين الثاني :



$L\ddot q + R\dot q + \frac{1}{C}q = E(t)$ —— (1)

$\ddot q + \frac{R}{L}\dot q + \frac{1}{LC}q = \frac{E(t)}{L}$

$\ddot q + 2\delta\dot q + \omega_0^2 q = \frac{E(t)}{L}$

$2\delta = \frac{R}{L}$ $\omega_0^2 = \frac{1}{LC}$

$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = E_0 e^{j\omega t}$ —— (2)

نفرض : $i = Ae^{j(\omega t+\varphi)}$

$\frac{di(t)}{dt} = j\omega A e^{j(\omega t+\varphi)}$

$\int i(t)dt = \frac{1}{j\omega}Ae^{j(\omega t+\varphi)}$

$Lj\omega Ae^{j(\omega t+\varphi)} + RAe^{j(\omega t+\varphi)} + \frac{1}{C}\frac{1}{j\omega}e^{j(\omega t+\varphi)}$
$= E_0 e^{j\omega t}$

$(Lj\omega + R + \frac{1}{C}\frac{1}{j\omega})i(t) = E(t)$

$(R + j(L\omega - \frac{1}{C\omega}))i(t) = E(t)$

$= B'\cos\varphi - jB'\sin\varphi$

$$\begin{cases}(\omega_0^2-\Omega^2)A = B'\cos\varphi \\ 2\delta\Omega A = -B'\sin\varphi\end{cases}$$

$$A = \frac{B'}{\sqrt{(\omega_0^2-\Omega^2)^2+(2\delta\Omega)^2}}$$

$$tg\,\varphi = \frac{-2\delta\Omega}{\omega_0^2-\Omega^2} \Rightarrow \varphi = arctg\frac{-2\delta\Omega}{\omega_0^2-\Omega^2}$$

$$\theta_p(t) = \frac{B'}{\sqrt{(\omega_0^2-\Omega^2)^2+(2\delta\Omega)^2}}\sin(\Omega + arctg\frac{-2\delta\Omega}{\omega_0^2-\Omega^2}$$

(2)

$T_m = \frac{1}{2}J\dot\theta^2 = \frac{1}{2}mL^2\dot\theta^2 = T$

$U_m = -\int_L^{L\cos\theta} mg\,dy$
$= -mgL(\cos - 1)$
$= -mgL(1 - \frac{\theta^2}{2} - 1) = mgL\frac{\theta^2}{2}$

$U_K = \frac{1}{2}Kx^2 = \frac{1}{2}K(\frac{L}{2}\theta)^2$
$= \frac{1}{2}K\frac{L^2}{4}\theta^2$

$U = mgL\frac{\theta^2}{2} + \frac{1}{8}KL^2\theta^2$

$D = \frac{1}{2}f\dot x_m^2$
$= \frac{1}{2}fL^2\dot\theta^2$

$$\frac{d}{dt}\left[\frac{\partial T}{\partial\dot\theta}\right] + \frac{\partial U}{\partial\theta} = \frac{\partial D}{\partial\dot\theta}$$

$mL^2\ddot\theta + mgL\theta + \frac{1}{4}KL^2\theta$
$= -fL^2\dot\theta + LF(t)$

$\Leftrightarrow mL^2\ddot\theta + fL^2\dot\theta + (mgL + \frac{1}{4}KL^2)\theta$
$= LF_0 \sin \Omega t$

$$Z(\omega) = R + j\left(L\omega - \frac{1}{C\omega}\right)$$

$$|Z(\omega)|\cdot|i| = |E|$$

$$= \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}\cdot A = E_0$$

$$i = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

3- الشدة الأعظمية توافق : $\left(L\omega - \frac{1}{C\omega}\right)^2 = 0$

$$L\omega - \frac{1}{C\omega} = 0 \Rightarrow L\omega = \frac{1}{C\omega}$$

$$\omega^2 = \frac{1}{CL} \Rightarrow \omega = \sqrt{\frac{1}{CL}} = \omega_0$$

ومنه : $Z(\omega) = R$

4-

$$y = \frac{\frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}}{\frac{E_0}{R}} = \frac{|i|}{|i_0|}$$

$$= \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$= \frac{R}{\sqrt{R^2 + R^2\left(\frac{L}{R}\omega - \frac{1}{RC\omega}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{L}{R}\omega - \frac{L}{R}\cdot\frac{1}{LC}\cdot\frac{1}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega_0}{2\sigma}\cdot\frac{\omega}{\omega_0} - \frac{\omega_0}{2\sigma}\cdot\omega_0\frac{1}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$

5-

$$= \frac{1}{\sqrt{1 + \frac{Q^2}{\omega_0^2}\left(\omega - \frac{\omega_0^2}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{Q^2}{\omega_0^2}\left(\frac{\omega^2 - \omega_0^2}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{Q^2}{\omega_0^2}\left(\frac{(\omega - \omega_0)(\omega + \omega_0)}{\omega}\right)^2}}$$

(annotations: $\omega - \omega_0 = \Delta\omega$, $\omega + \omega_0 \approx 2\omega$)

$$= \frac{1}{\sqrt{1 + 4x^2}}$$

6-

$$y = \frac{|i|}{|i_0|} = \frac{\frac{|i_0|}{\sqrt{2}}}{|i_0|} = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{1 + 4x^2}} \qquad 1 + 4x^2 = 2$$

$$\Rightarrow x^2 = \frac{1}{4} \qquad x = \frac{1}{2} \;;\; x = -\frac{1}{2}$$

$$-x = -\frac{1}{2}$$

$$x = Q\frac{\Delta\omega}{\omega_0} = Q\,\frac{\omega_0 - \omega_0}{\omega_0} = \frac{1}{2}$$

$$\omega_2 = \frac{1}{2}\frac{\omega_0}{Q} + \omega_0$$

$$= \omega_0\left(1 + \frac{1}{2Q}\right)$$

$$x = \frac{1}{2}$$

$$x = \frac{Q(\omega_0 - \omega)}{\omega_0} = \frac{1}{2}$$

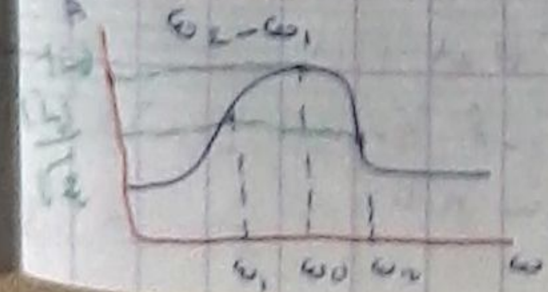
$$\omega_1 = -\left(\frac{1}{2}\frac{\omega_0}{Q} - \omega_0\right) = \omega_0\left(1 - \frac{1}{2Q}\right)$$

7-

$$B = \omega_2 - \omega_1 = \omega_0\left(1 + \frac{1}{2Q} - 1 + \frac{1}{2Q}\right)$$

$$B = \frac{\omega_0}{Q} \Rightarrow Q = \frac{\omega_0}{B}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$



السلسلة (05) =

التمرين الخامس =

$$3m\ddot{x} = -T\sin\alpha + T\sin\beta$$

$$8m\ddot{y} = -T\sin\beta - T\sin\gamma$$

الاهتزاز بزوايا صغيرة جدا.

$$\sin\alpha \simeq tg\,\alpha = x/a$$

$$\sin\beta \simeq tg\,\beta = \frac{y-x}{a}$$

$$\sin\gamma \simeq tg\,\gamma = \frac{y}{a}$$

$$\begin{cases} 3m\ddot{x} = -T\left(\frac{x}{a}\right) + T\,\frac{(y-x)}{a} \\ 8m\ddot{y} = -T\frac{(y-x)}{a} - T\left(\frac{y}{a}\right) \end{cases}$$

$$3m\ddot{x} = -\frac{2}{a}Tx + \frac{T}{a}y$$

$$8m\ddot{y} = -\frac{2T}{a}y + T/a\,x$$

$$\begin{cases} 3m\ddot{x} + 12mK^2x - 6mK^2y = 0 \\ 8m\ddot{y} + 12mK^2y - 6mK^2x = 0 \end{cases}$$

$$\begin{cases} \ddot{x} + 4K^2x - 2K^2y = 0 \\ \ddot{y} + 3/2K^2y - \frac{3}{4}K^2x = 0 \end{cases}$$

نضع : (الحلول الذاتية)

$$x = A\sin\omega t$$

$$y = B\sin\omega t$$

$$\ddot{x} = -A\omega^2\sin\omega t = -\omega^2 x$$

$$\ddot{y} = -B\omega^2\sin\omega t = -\omega^2 y$$

$$\begin{cases} (-A\omega^2 + 4K^2A - 2K^2B)\sin\omega t = 0 \\ (-B\omega^2 + 3/2K^2B - 3/4K^2A)\sin\omega t = 0 \end{cases}$$

$$A(-\omega^2 + 4K^2) - 2K^2B = 0$$

$$-\frac{3}{4}K^2A + (-\omega^2 + 3/2K^2)B = 0$$

$$\begin{vmatrix} -\omega^2+4K^2 & -2K^2 \\ -\frac{3}{4}K^2 & -\omega^2+3/2K^2 \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}.$$

حتى لا يكون لدينا الحل معدوما :

$$A = B = 0$$

$$\Rightarrow \det = 0$$

$$\Rightarrow [(-\omega^2+4K^2)(-\omega^2+3/2K^2)] - [(-2K^2)(-3/4K^2)] = 0$$

$$\omega^4 - \frac{11}{2}K^2\omega^2 + 9/2K^4 = 0$$

نضع : $\omega^2 = Z$

$$Z^2 - 11/2K^2Z + 9/2K^4 = 0$$

$$\Delta = (-11/2K^2)^2 - 4(1)(9/2K^4)$$

$$= \frac{121}{4}K^4 - 18K^4 = \frac{49}{4}K^4$$

$$\sqrt{\Delta} = \frac{7}{2}K^2$$

$$Z_1 = K^2 \qquad Z_2 = 9/2K^2$$

$$\omega_1 = K \qquad \omega_2 = 3/\sqrt{2}\,K$$

(حلول الحركة) ← الشكل الاهتزازي للنظام

$$x = A_1\sin(\omega_1 t + \varphi_1) + A_2\sin(\omega_2 t + \varphi_2)$$

$$y = B_1\sin(\omega_1 t + \varphi_1) + B_2\sin(\omega_2 t + \varphi_2)$$

أنماط الحركة :

أ) $\omega = \omega_1 = K$

$$(-K^2 + 4K^2)A - 2K^2B = 0$$

$$(3K^2)A - 2K^2B = 0$$

$$3K^2A = 2K^2B \Rightarrow B_1 = 3/2\,A_1$$

ب) $\omega = \omega_2 = 3/\sqrt{2}\,K$

$$(-9/2K^2 + 4K^2)A - 2K^2B = 0$$

$$-\frac{K^2}{2}A = 2K^2B \Rightarrow B_2 = -1/4\,A.$$

$U_k = \frac{1}{2}K(x - L\theta)^2$

$U = Mg\frac{L}{4}\theta^2 + \frac{1}{2}K(x - L\theta)^2$

$= Mg\frac{y^2}{4L} + \frac{1}{2}K(x - y)^2$

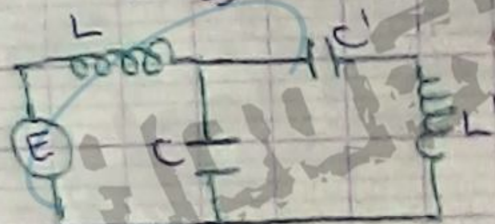
$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) + \frac{\partial U}{\partial x} = F(t)$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) + \frac{\partial U}{\partial y} = 0$

$m\ddot{x} + K(x - y) = F_0 \sin\omega t$

$\frac{1}{3}M\ddot{y} + \frac{1}{2}Mg\,y - K(x - y) = 0$

وجود ارتباط مرونتي



-4 $m\ddot{x} + Kx - Ky = F_0 \sin\omega t$

$2m\ddot{y} + 2Ky - Kx = 0$

$x = A\sin\omega t \Rightarrow \ddot{x} = -A\omega^2 \sin\omega t$

$y = B\sin\omega t \Rightarrow \ddot{y} = -B\omega^2 \sin\omega t$

$(-m\omega^2 A + KA - KB)\sin\omega t = F_0 \sin\omega t$

$(-2m\omega^2 B + 2KB - KA)\sin\omega t = 0$

$\begin{cases}(-m\omega^2 + K)A - KB = F_0 \\ -KA + 2(-m\omega^2 + K)B = 0\end{cases}$

$\begin{vmatrix}-m\omega^2 + K & -K \\ -K & 2(-m\omega^2 + K)\end{vmatrix}\begin{vmatrix}A \\ B\end{vmatrix} = \begin{vmatrix}F_0 \\ 0\end{vmatrix}$

-5 $\begin{vmatrix}-m\omega^2 + K & -K \\ -K & 2(-m\omega^2 + K)\end{vmatrix}\begin{vmatrix}A \\ B\end{vmatrix} = \begin{vmatrix}0 \\ 0\end{vmatrix}$

ومنه :

$\begin{cases}x = A_1\sin(\omega_1 t + \varphi_1) + A_2\sin(\omega_2 t + \varphi_2) \\ y = \frac{3}{2}A_1\sin(\omega_1 t + \varphi_1) - \frac{1}{4}A_2\sin(\omega_2 t + \varphi_2)\end{cases}$

-2 $\begin{cases}\dot{x} = A_1\omega_1\cos(\omega_1 t + \varphi_1) + A_2\omega_2\cos(\omega_2 t + \varphi_2) \\ \dot{y} = \frac{3}{2}A_1\omega_1\cos(\omega_1 t + \varphi_1) - \frac{1}{4}A_2\omega_2\cos(\omega_2 t + \varphi_2)\end{cases}$

ومنه :

$x(0) = x_0 \Rightarrow A_1\sin\varphi_1 + A_2\sin\varphi_2 = x_0$

$y(0) = 0 \Rightarrow \frac{3}{2}A_1\sin\varphi_1 - \frac{1}{4}A_2\sin\varphi_2 = 0$

$\dot{x}(0) = 0 \Rightarrow A_1\omega_1\cos\varphi_1 + A_2\omega_2\cos\varphi_2 = 0$

$\dot{y}(0) = 0 \Rightarrow \frac{3}{2}A_1\omega_1\cos\varphi_1 - \frac{1}{4}A_2\omega_2\cos\varphi_2 = 0$

$\varphi_1 = \varphi_2 = \pi/2$

-3 $A_1 + A_2 = x_0$

$\frac{3}{2}A_1 - \frac{1}{4}A_2 = 0 \Rightarrow \frac{3}{2}A_1 = \frac{1}{4}A_2$

$\Rightarrow A_2 = 6A_1$

$A_1 = x_0/7 \quad , A_2 = 6/7\, x_0$

$\varphi_1 = \varphi_2 = \pi/2$

$x = \frac{x_0}{7}\sin(Kt + \frac{\pi}{2}) + \frac{6}{7}x_0\sin(\frac{3}{\sqrt{2}}Kt + \frac{\pi}{2})$

$y = \frac{3}{14}x_0\sin(Kt + \frac{\pi}{2}) - \frac{6}{18}x_0\sin(\frac{3}{\sqrt{2}}Kt + \frac{\pi}{2})$

التمرين الخامس :

-1 $T = T_M + T_m$

$T_M = \frac{1}{2}(\frac{1}{3}ML^2)\dot{\theta}^2$

$T_m = \frac{1}{2}m\dot{x}^2$

$T = \frac{1}{6}M\dot{y}^2 + \frac{1}{2}m\dot{x}^2$

-5 $U = U_m + U_k$

$U_m = \int_{L/2}^{\frac{L}{2}\cos\theta} Mg\,dy = -Mg\frac{L}{2}(\cos\theta - 1)$

$= -Mg\frac{L}{4}\theta^2$

$\det = 0 \Rightarrow (-m\omega^2+k)2(-m\omega^2+k) - (k)(-k) = 0$

$\Rightarrow 2(-m\omega^2+k)^2 - k^2 = 0$

$\Rightarrow [\sqrt{2}(-m\omega^2+k)]^2 - k^2 = 0$

$\Rightarrow [-\sqrt{2}m\omega^2+(\sqrt{2}-1)k][-\sqrt{2}m\omega^2+(\sqrt{2}+1)k] = 0$

$\omega_1^2 = \frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{k}{m} \qquad = \frac{\sqrt{2}+1}{\sqrt{2}} \cdot \frac{k}{m}$

$\begin{vmatrix} (-m\omega^2+k) & -k \\ -k & 2(-m\omega^2+k) \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} F_0 \\ 0 \end{vmatrix}$ — 6

$A = \dfrac{2F_0(-m\omega^2+k)}{[-\sqrt{2}m\omega^2+(\sqrt{2}-1)k][-\sqrt{2}m\omega^2+(\sqrt{2}-1)k]}$

$B = \dfrac{F_0 \cdot k}{[-\sqrt{2}m\omega^2+\sqrt{2}-1)k][-\sqrt{2}m\omega^2+\sqrt{2}+1)k]}$

